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THE SUSTAINMENT DYNAMO REEXAMINED: NONLOCAL ELECTRICAL CONDUCTIVITY OF PLASMA IN A STOCHASTIC MAGNETIC FIELD

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The "plasma dynamo" is both an intriguing and a practical concept. The intrigue derives from attempting to explain naturally occurring[1] and man-made[2,3] plasmas whose strong field-aligned currents $j_\parallel$ apparently disobey the most naive Ohm's law $j_\parallel = \sigma_\parallel E_\parallel$. The practical importance derives from the dynamo's role both in formation and in sustainment of reversed-field pinch (RFP)[2] and Spheromak[3] fusion plasmas. We will examine certain features of the documented quasi-steady discharges[2] on ZT-40M, an RFP in apparent need[4] of a sustainment dynamo. We will show that the tail electrons (which carry $j_\parallel$) are probably wandering (along stochastic $\vec{B}$-field lines) over much of the minor radius in one mean-free-path. This will void any local Ohm's law, whether naive ($j_\parallel = \sigma_\parallel E_\parallel$) or containing additional terms (such as the $\langle\vec{v}\times\vec{B}\rangle_\parallel$ of nonlinear dynamo theory). Instead, we will show that observed quasi-steady RFP discharges in ZT-40M are explainable in simple terms ($f = ma$) of electron-momentum diffusion in a stochastic field, using a stochasticity inferred from observed τ_{Ee}. We will then present results of a formal model of this momentum diffusion. The model predicts the key observed anomalies of sustained RFP behavior (excess loop resistance; slower-than-classical current decay) in terms of electron dynamics in a stochastic magnetic field. Absent from our model are the usual turbulent-dynamo concepts: magnetic-helicity conservation, mode-mode interactions, relaxation, wavenumber cascades, etc.

Quasi-steady discharges that defy a naive Ohm's law[4] have been reported[2] on ZT-40M. Their parameter regime is low density ($n \lesssim 2 \times 10^{19}\mathrm{m}^{-3}$), high temperature ($T_e \lesssim 150$ eV), and electron heat-loss time $\tau_{Ee} \simeq 10^{-4}$s. At moderate pinch parameter ($\theta \lesssim 1.5$) these RFP discharges show very little poloidal variation of the reversed toroidal field $[B_\phi(a)]$ apart from the factor $1/R$: $[\Delta B_z(a)/B_z(a)]_{rms} \lesssim 0.1$ and $[\Delta B_z(a)/B_\theta(a)]_{rms} \lesssim 0.01$. This observed laminarity does not appear to be consistent with the sustainment dynamo's properties seen in MHD calculations by Sykes and Wesson[5] and by Aydemir and Barnes,[6] both of which calculations predict[7,8] such large-scale poloidal asymmetry that $B_z(a)$ is not even everywhere reversed, i.e., $[\Delta B_z(a)/B_z(a)] \sim 1$.

Rechester and Rosenbluth[9] showed that a typical Tokamak can be driven stochastic (i.e., islands overlap everywhere) with $(B_r^{local}/B_o)_{rms} \gtrsim 10^{-5}$ if a wavenumber spectrum populated out to $k_\perp\rho_{ci} \simeq 1$ is assumed. Repeating their exercise for a typical RFP indicates $(B_r^{local}/B_o)_{rms} \gtrsim 10^{-4}$ would produce stochasticity. The point we make is that even such a level is undetectable, so that Ockham's Razor would favor stochasticity as the cause of observed, nonradiative electron heat loss ($\tau_{Ee} \simeq 10^{-4}$s) in ZT-40M.

If we assume ZT-40M is stochastic, then the electron heat diffusivity[9] required to cause τ_{Ee} can be used to estimate the magnetic field-line diffusivity D_F. Krommes et al.,[10] suggest that this estimate will be a lower bound for D_F. If we write $\tau_{Ee} \simeq a^2/D_e$, the electron-heat diffusivity (with $a = 0.2$ m) is $D_e \simeq 4 \times 10^2\mathrm{m}^2\mathrm{s}^{-1}$. An upper bound[10] on the stochasticity-induced electron-heat diffusivity[9] is $D_e \simeq v_{Te}D_F$. Using $T_e \simeq 200$ eV so that $v_{Te} \simeq 6 \times 10^6\mathrm{ms}^{-1}$, we get $D_F \simeq 7 \times 10^{-5}$m as a lower bound on the magnetic-field-line diffusivity.

How far does an electron wander during one mean-free-path across the flux surfaces, if indeed $D_F \simeq 7 \times 10^{-5}$m? The most probable electron ($v = v_{Te}\sqrt{2} \equiv v_o$) has a mean-free-path (in a Lorentz plasma with $Z = 1$, $n = 2 \times 10^{19}\mathrm{m}^{-3}$, and $T_e = 200$ eV) $\lambda_o = 20$ m. The more relevant number, though, is λ averaged over j, and this can be shown[11] to be $\lambda_j \equiv \int\lambda dj/\int dj = 20\ \lambda_0$ for a Lorentz plasma owing to the weighting of suprathermal electrons in carrying j. Using $\lambda_j = 400$ m, we obtain an electron wander $(\Delta x)_j = (2D_F\lambda_j)^{1/2} = 0.3$ m as a lower bound. Thus, in one mean-free-path the j-weighted electron radial wander is similar to the plasma radius!

Consider a slab-geometry RFP with x the normal to "flux surfaces" (like r in a cylinder). The configuration is sustained by a steady, uniform applied E_z. The local magnetic-field-aligned electric field is $E_\parallel(x) = E_z B_z(x)/B$. The average gradient length $E_\parallel/(\partial E_\parallel/\partial x)$ in an RFP will be smaller than a. Thus in ZT-40M, tail electrons wander all over the $E_\parallel$-gradient in one mean-free-path. This voids a local Ohm's law. More importantly, it suggests that RFP sustainment on ZT-40M may be due to export of electron field-aligned momentum from the core (where $E_\parallel > j_\parallel/\sigma_\parallel$) to the outer region (where $E_\parallel \lesssim 0 < j_\parallel/\sigma_\parallel$).

We have recently developed[11] a formal procedure for treating electron-momentum export down the $E_\parallel$-gradient. The treatment is facilitated by some simplifying assumptions (none of which, though, is required for the basic mechanism to be viable):

1. The plasma is isothermal and isodense, and $f^{(0)}(\vec{v})$ is a Maxwellian.
2. Slab-geometry is employed, and $|\vec{B}|$ is uniform.
3. Coulomb scattering is approximated by electron collisions only with massive ions (Lorentz gas).
4. The applied electric field is weak: $E_\parallel \ll E_c$, where $E_c \equiv$ critical (runaway) field.[12]
5. $L_F \ll \lambda$ where L_F is the (Kolmogorov) correlation length[9] and λ is the electron mean-free-path.

In these conditions we have obtained[11] the following results:

First: The perturbation $f^{(1)}(\vec{v},x)$ in the electron distribution function is laminar, depending on x (the normal to "flux surfaces") but not on y or z.

Second: The perturbation $f^{(1)}(\vec{v},x)$ is purely odd in $\cos\theta$ (where θ is the angle between $\vec{v}$ and $\vec{B}$); this leads to export of field-aligned momentum, <u>but not of electron number density</u>, down the $E_\parallel$-gradient.

Third: The spatial gradient $\partial f^{(1)}(\vec{v},x)/\partial x$ causes a Fick's Law flux $-D_e\ \partial f^{(1)}(\vec{v},x)/\partial x$, which carries the electron momentum exported down the $E_\parallel$-gradient.

Fourth: For each electron velocity $\vec{v}$, $f^{(1)}(\vec{v},x)$ is a solution of a separate Boltzmann equation:

$$f^{(1)}(\vec{v},x) = -\frac{E_\parallel(x)}{E_c}\left(\frac{v}{v_0}\right)^4 \cos\theta f^{(0)}(\vec{v}) + 2\lambda_0\left(\frac{v}{v_0}\right)^4 |\cos\theta| \frac{\partial}{\partial x}\left[D_F(x)\ \frac{\partial f^{(1)}(\vec{v},x)}{\partial x}\right]. \quad (1)$$

The first term on the rhs of Eq. (1) is the local Spitzer-Härm[13] Lorentz-gas solution. The second term on the rhs is (minus) the divergence of the Fick's law flux down the spatial gradient of $f^{(1)}(\vec{v},x)$. The $(v/v_0)^4|\cos\theta|$ weighting is caused by the mean-free-path's dependence on $\vec{v}$.

We solve Eq. (1), with $E_\parallel(x)$ and $D_F(x)$ profiles as inputs, at each of 39 velocities (3 angles, θ, at each of 13 speeds, v). The solutions are multiplied by $-ev\cos\theta$ and integrated $d\vec{v}$ with splines to give $j_\parallel(x)$. The contrived boundary condition at the wall is $\partial f^{(1)}/\partial x_{x=a} = 0$, corresponding to zero momentum export from the plasma to the wall. The $E_\parallel(x)$ profile shape is affected by the $j_\parallel(x)$ result, because $j_\parallel(x)$ controls the magnetic field orientation (via Ampere's law), and $E_\parallel(x) = E_z B_z(x)/B$. Thus we iterate the solution of Eq. (1), at each step using an updated $E_\parallel(x)$ profile, until the current $j_\parallel(x)$ satisfies both f = ma [Eq. (1)] and Ampere's law.

The parameters which we may choose are $\lambda_0 D_F/a^2$ (characterizing the electron wander) and $j_\parallel(0)/B$ (corresponding to how hard we push the system). In order to compare with RFP phenomenology we may use $B_y(a)/\langle B_z\rangle$

(corresponding to the pinch parameter, θ) as the second parameter instead of $j_\parallel(0)/B$.

A self-consistent solution with uniform diffusivity ($\lambda_o D_F/a^2 = 0.05$) and pinch parameter $B_y(a)/\langle B_z\rangle = 2.10$ is shown in Fig. 1. The $E_\parallel(x)$ profile has the same shape as the $B_z(x)$ profile. Despite the $E_\parallel(x)$ profile's sign reversal (at $x \simeq 0.8a$), the field-aligned current $j_\parallel(x)$ is almost flat, and never reverses sign. The microscopic reason for this is the spatially diffused profiles of $f^{(1)}(\vec{v},x)$, shown in Fig. 2. For almost-perpendicular ($\cos\theta = 0.3$) and low-speed ($v/v_o = 0.8$) velocities, the conduction $f^{(1)}(\vec{v},x)$ closely resembles $E_\parallel(x)$ in shape (Fig. 2, top). However, field-aligned ($\cos\theta = 1.0$) suprathermal electrons ($v/v_o > 1$) have more diffused $f^{(1)}(\vec{v},x)$ profiles (Fig. 2, bottom).

In Fig. 3 we show the "resistive anomaly," that is, the ratio of E_z to $j_\parallel(0)/\sigma$, where $\sigma \equiv$ nominal local Ohm's-law conductivity. Our resistive anomaly is understated because we do not consider electron-momentum loss to the wall.

An "F-Θ diagram" for slab geometry is shown in Fig. 4, using various spatially uniform diffusivities $\lambda_o D_F/a^2$. The extreme case ($\lambda_o D_F/a^2 = \infty$) would be called "fully relaxed," and the others "partially relaxed" in dynamo parlance. In our theory of nonlocal conductivity, however, "relaxation" plays no role; instead, the F-Θ trajectory is controlled by the range of electron wander, measured by $\lambda_o D_F/a^2$.

We have also calculated RFP states for tapered profiles of $D_F(x)$, in which D_F is high on axis (x=0) but falls to the edge (x=a). [This $D_F(x)$ profile may be appropriate to RFP experiments owing to the tendency of the nearby conducting shell to reduce B_r-fluctuations near the edge.] We find that the $j_\parallel(x)$ profile responds by also becoming reduced at the edge. This may account for the "Modified" (i.e., tapered at edge) current profiles inferred in experiments.[2]

Finally, the nonlocal-conductivity model offers some insight on the time scale required for an RFP discharge to relax following a step change in some boundary condition (e.g., toroidal flux or toroidal voltage): Although the model described above is steady-state, it is clear that the $j_\parallel(x)$ profile can relax no more quickly than a j-weighted electron-ion collision time.

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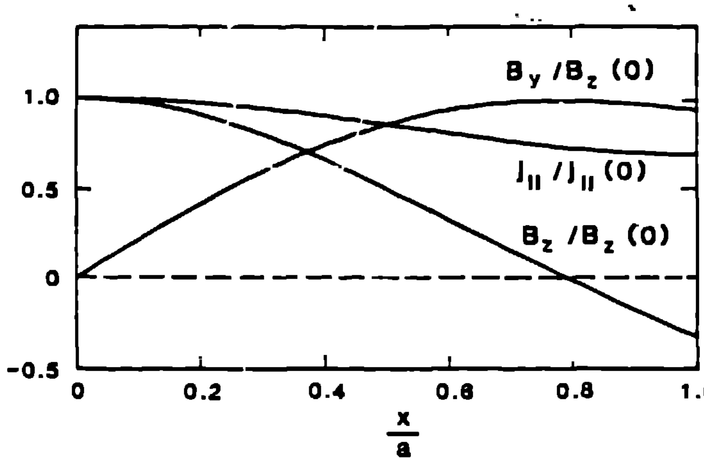

Fig. 1. Normalized profiles of magnetic fields and field-aligned current density for uniform diffusivity.

$$\frac{\lambda_0 D_F}{a^2} = 0.05$$

$$\frac{B_y(a)}{\langle B_z \rangle} = 2.10$$

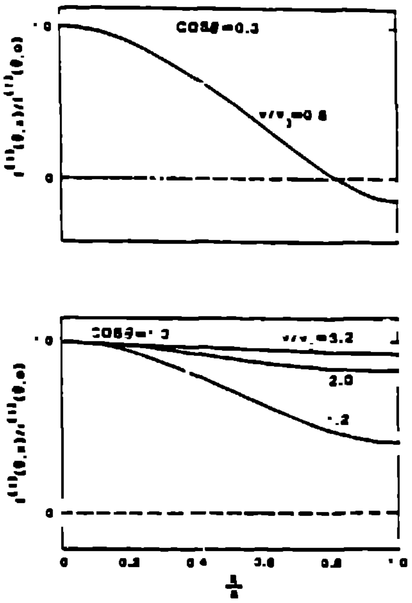

Fig. 2. Normalized profiles of electron distribution-function perturbation for four velocities, in conditions of Fig. 1.

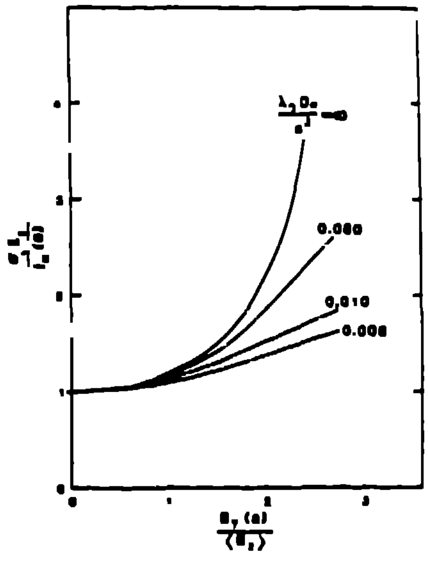

Fig. 3. Resistive anomaly factor versus pinch parameter, for various diffusivities.

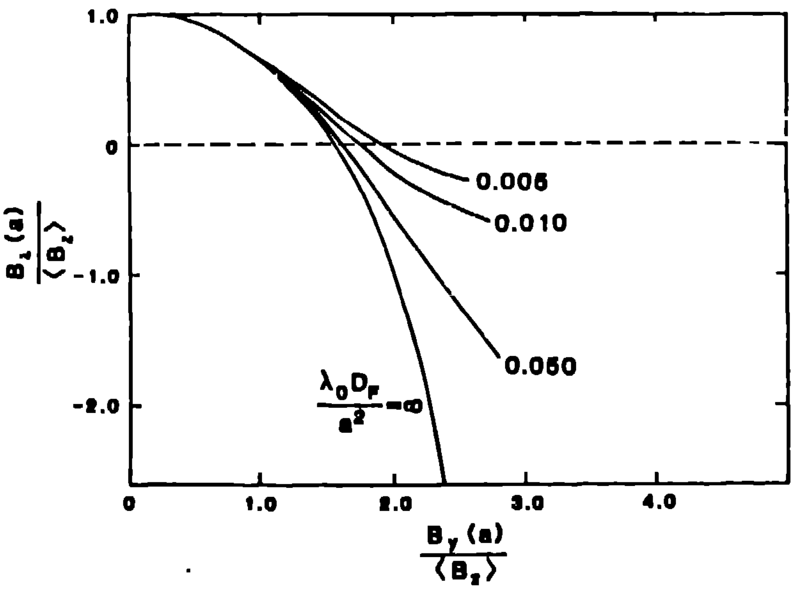

Fig. 4. F-Θ trajectories for various diffusivities, in slab-geometry.